**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANS** - Mean service time ($\mu$) = 45 minutes

Standard deviation ($\sigma$) = 8 minutes

Work begins 10 minutes after drop-off

Car should be ready within 1 hour from drop-off

Let X be the time required for servicing transmissions. We want to find P(X > 60), where X ~ N(45, 8^2).

Using z-score:

Using z-table or calculator, we find the probability corresponding to z = 1.875, which is approximately 0.0301.

So, the probability that the service manager cannot meet his commitment is approximately 0.0301.

Answer: Option A. 0.3875

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS** - Mean age ($\mu$) = 38

Standard deviation ($\sigma$) = 6

A. True/False: More employees are older than 44 than between 38 and 44.

True. We can infer this because the mean (38) is less than 44, and the distribution is normal.

B. True/False: A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

False. We cannot infer the number of employees under 30 without knowing the proportion of employees in that age group.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS -** X1 ~ N(μ, σ^2)

X2 ~ N(μ, σ^2)

We need to discuss the distributions and parameters of 2X1 and X1 + X2.

For 2X1:

Distribution: Normal with mean 2μ and variance 4σ^2.

Parameters: Mean = 2μ, Variance = 4σ^2.

For X1 + X2:

Distribution: Normal with mean 2μ and variance 2σ^2.

Parameters: Mean = 2μ, Variance = 2σ^2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**ANS -** X ~ N(100, 20^2)

Probability of values between a and b is 0.99.

We need to find symmetric values a and b such that P(a < X < b) = 0.99.

Using z-score:

From the z-table, for 0.99 probability, the z-score is approximately ±2.576.

So, the symmetric values are approximately 48.48 and 151.52.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**ANS**- Profit1 ~ N(5, 3^2) in $ Million

Profit2 ~ N(7, 4^2) in $ Million

A. Rupee range containing 95% probability for the total profit of the company:

Convert profit distributions to Rupees using $1 = Rs. 45.

Calculate total profit distribution as Profit1 + Profit2.

Find the 2.5th and 97.5th percentiles of the total profit distribution.

B. 5th percentile of profit (in Rupees) for the company:

Convert profit distribution to Rupees using $1 = Rs. 45.

Find the 5th percentile of the total profit distribution.

C. Division with a larger probability of making a loss:

Calculate the probability of making a loss for each division separately.

Compare the probabilities.